Stellar Structure and Evolution
Final Exam (Re-examination)
6 February 2009

DIRECTIONS: ALLOW 3 HOURS (about 30 minutes per question). Closed book and closed notes. Answer all questions. All questions are worth the same value. Some questions depend at least partly on the answers to previous questions. Keep answers reasonably short and to the point, but show ALL work.

Possibly useful constants in cgs units (to two significant figures):

$$\begin{aligned} R_g &= 8.3 \times 10^7 \\ G &= 6.7 \times 10^{-8} \\ k_B &= 1.4 \times 10^{-16} \\ m_{\rm H} &= 1.7 \times 10^{-24} \end{aligned}$$

1. The stellar structure equations.
   (a) Write down the four stellar structure equations.
   (b) Specify the boundary conditions, and explain why the number of boundary conditions you use is correct.
   (c) Write down the order-of-magnitude equivalents of these equations (for example, R/M and $\bar{P}/M$).
2. The mass–luminosity relation.
   (a) Assume that the equation of state is ideal gas. Taking care to keep track of μ in your equations, derive an order-of-magnitude estimate of the luminosity as a function of stellar mass. Show that it is independent of the nuclear energy generation rate ϵ. [Hint: use your answers from 1(c) above; you might also need to rewrite the equation for radiative transfer in an Eulerian rather than Lagrangian form and make an assumption about the magnitude of dT/dr.]
   (b) Now assume that the opacity is electron scattering. What is the scaling of luminosity with mass?
   (c) Now assume that the opacity is Kramer's and that the equation of state is again ideal gas. Derive a new scaling of luminosity with mass, assuming that such stars are fully radiative.
   (d) Now assume that radiation pressure dominates, and derive a new scaling. (Hint: what is the opacity source in stars like this?)
   (e) Do stars' lifetimes increase or decrease with increasing mass? Why? What is the scaling of lifetime with mass?
   (f) How do lifetimes, surface luminosities, and effective temperatures scale with metal abundance Z? With helium abundance Y? Why?
3. Draw a large Hertzsprung-Russell (HR) diagram and label the axes clearly. What measurements have to be made to determine the ordinate ($\log L$) for a particular star? Indicate where in the diagram the following objects would lie:
   (a) the present Sun

(b) a red giant undergoing the helium flash

(c) a pre-main-sequence star of about 0.2 $M_\odot$ at the bottom of its Hayashi track

(d) a main-sequence star in which radiation pressure is important

(e) a star of 5 $M_\odot$ with solar metallicity burning helium in a non-degenerate core.

For each of these cases, what kinds of energy transport are occuring, and where in the star are they happening?

4. The radiative gradient is given by

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2}$$

   (a) Combine this equation with the equation for hydrostatic equilibrium to get an expression for dP/dT.

   (b) Assume an ideal gas equation of state, no radiation pressure, zero boundary conditions, and an opacity from electron scattering only. Integrate the equation to find $P(T)$ in the envelope (which is defined by $L_r = L$, $M_r = M$).

   (c) Test this distribution to find out whether it is stable or unstable to convection.

5. Briefly explain the meaning and importance of the following terms:

   (a) free-free transition

   (b) local thermodynamic equilibrium

   (c) mixing length

   (d) Hayashi limit

   (e) mean lifetime of a nucleus

6. Consider a contracting star in hydrostatic equilibrium, which stops contracting when the temperature reaches a critical value T_c required for hydrogen burning.

   (a) What is the pressure P_c at the center of the star due to gravitational forces? You can assume a constant density ρ_c.

   (b) What is the ideal gas pressure P_c at the center of the star?

   (c) Balance these pressure forces to show that the greater the mass of the star, the smaller its density in the center when T_c is reached. Hint: eliminate R in favor of M and ρ_c.

   (d) Now use the rough criterion for the transition from ideal gas to (non-relativistic) electron degeneracy, $\rho = 10^{-19.5}\left(\frac{k_B}{\mu m_H}\right)^{3/2}\mu_e^{5/2}T^{3/2}$, to estimate the critical mass (to order of magnitude) at which the contraction is halted by degeneracy instead of hydrogen burning. Assume $T_c \approx 5 \times 10^6$ K and $X = 0.7$.

   (e) How could you improve this estimate without actually calculating stellar models?

Question 2 (Chapter 6)
A pressure sensor with built-in current transmitter (characterized by its Norton equivalent circuit: current source i_N with shunt resistance R_N) is read out by a recorder with load resistance R_L. The capacitive coupling to a nearby power cable and earth plane (same as the recorder ground) can be described by the parasitic capacitors C_1, C_2, C_3, and C_4, as shown in Figure 2.

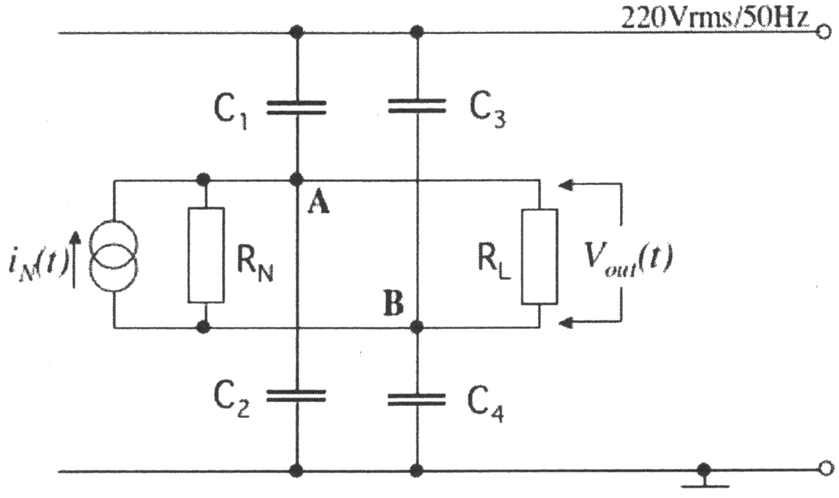

Figure 2. Equivalent circuit of pressure transducer and recorder, showing capacitive coupling to power cable and earth plane. R_N=100 kΩ, R_L=1 kΩ, C_1=50 pF, C_2=55 pF, C_3=80 pF, C_4=85 pF

Assume for the moment that $i_N = 0$ A and $R_N = R_L = \infty\ \Omega$.

a. Derive an expression for the common mode voltage (rms) seen by the recorder, and evaluate it numerically.

b. Derive an expression for the series mode voltage (rms) seen by the recorder, and evaluate it numerically.

The current is given by $i_N(t) = c\cdot sin(\omega t)$, with c = 6.3 mA and ω=1 kHz.

c. Ignoring the influence of the capacitive coupling, calculate the rms signal registered by the recorder.

d. Discuss strategies for reduction of the capacitive coupling (considering the physical characteristics that are at the origin of the capacitive coupling).

e. Estimate the signal-to noise ratio of the measurement, provided that the recorder has a CMRR=60 dB and that the sole source of noise is the capacitive coupling to the power cable.